

Testing Expected Utility in the Presence of Errors

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Abstract:

One possible conclusion from recent experimental research on decision making under risk is that observed behaviour can be reasonably accounted for by expected utility plus error term. This conclusion implies that the violation rate of expected utility should decrease if errors are excluded. This paper reports on an experiment which investigates this implication by presenting the same choice problems to subjects three times. The results show that the exclusion of errors leads to a significant reduction of the violation rate for most of the subjects and most of the choice problems. This observation can be regarded as supporting evidence for expected utility plus error term.

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1 Introduction

The common consequence effect and the common ratio effect, both introduced by Allais (1953), are the most prominent and most investigated violations of expected utility (EU). They have motivated the development of alternative theories of choice under risk which are able to account for the observed behavioural patterns. Nowadays a large number of alternative theories exist (cf. Starmer (2000), Sugden (2004), and Schmidt (2004) for surveys), and naturally the question arises which theory can explain observed behaviour best.

Many studies have addressed this question; most prominent seem to be those of Harless and Camerer (1994) and Hey and Orme (1994). Since EU and its alternatives are deterministic theories, but observed choices are stochastic both papers integrated an error term into their estimations. This fact has aroused interest in a general discussion of the role of errors in decision making under risk. The discussion can be disentangled into two issues. The first issue concerns the modelling of the stochastic component (Harless and Camerer 1994; Hey and Orme 1994; Camerer and Ho 1994; Hey 1995; Loomes and Sugden 1995; Loomes and Sugden 1995) and corresponding experimental tests (Carbone 1997; Ballinger and Wilcox 1997; Loomes and Sugden 1998; Carbone and Hey 2000; Buschena and Zilberman 2000; Loomes, Moffat and Sugden 2002). The second issue concerns the performance of EU and its alternatives for given specifications of the error term. In this context, Hey (1995), building upon the results of Hey and Orme (1994), arrives at the following conclusion:

“It may be the case that these further explorations may alter the conclusion to which I am increasingly being drawn: that one can explain experimental analyses of decision making under risk better (and simpler) as EU plus noise – rather than through some higher level functional – as long as one specifies the noise appropriately.”

This conclusion is reinforced by the results of Buschena and Zilberman (2000) which show that, under heteroscedastic error terms, the alternative theories do not offer a “statistically significant improvement in predictive power over EU”.

These conclusions are obviously in conflict with the high violation rates of EU observed in the common consequence and common ratio effects, and lead us to the following question: can EU - plus an appropriate error term - be the correct representation of preferences although there exist choice problems for which most subjects violate EU? Answering this question with yes and assuming that EU is the correct model obviously implies that the observed violations of EU are due to errors. The goal of the present paper is to analyse whether this is true.

For this purpose we have designed and run an experiment, in which subjects have to respond to identical binary choice problems on three different days. If a subject makes the same choice on all three days we say the observed choice is repetition-consistent, otherwise we say the observed choice is repetition-inconsistent. We propose a simple model based on the assumption that individuals have a deterministic preference ordering over lotteries which can be represented by EU. According to the model, choice must not always be consistent with this preference ordering, because with some probability individuals commit errors. If the error probability is not too high and the observed choice is repetition-consistent, it is very likely that this choice reflects the true preference of the individual. Under the assumption that EU plus error term is the correct representation of true preferences, we show that the violations of EU are less likely when choices are repetition-consistent than when choices are repetition-inconsistent. On the contrary, if true preferences are in conflict with EU for a given choice problem, violations of EU are more likely if choice is repetition-consistent than when it is repetition-inconsistent. Therefore, our experimental data allow us to test EU plus error term against any alternative model that has, in the absence of errors, different predictions than EU for the considered choice problems. Since we observe a lower violation rate of EU in repetition-consistent choice than in repetition-inconsistent choice we conclude that EU plus error term describes the data better than these alternatives. The experimental design is presented in the next section. Section 3 discusses our theoretical framework and explains our hypothesis in more detail. The results are presented in Section 4, and Section 5 contains some concluding remarks.

2 Experimental Design

The experiment was conducted at the Centre of Experimental Economics at the University of York with 24 participants. Each participant had to attend five separate sessions, A, B, C, D, and E, on five different days. After a subject had completed all five sessions, one question of one session was randomly selected and played out for real. The average payment to the subjects was £34.17 with £80 being the highest and £0 being the lowest payment.

In each of the five sessions subjects were presented the same 30 lottery pairs, 28 risky ones and two ambiguous ones (which are not analysed in this paper). All risky lotteries were composed of the four consequences £0, £10, £30, £40. The probabilities of these consequences are recorded in Appendix 1 for all 28 lottery pairs. Note that in each pair the left lottery is safer than the right lottery, though in the experiment the left-right juxtaposition was randomised. The lotteries were presented as segmented circles on the computer screen. If a subject received a particular lottery as a reward he or she had to spin a wheel on the corresponding circle. The amount won was then determined by the segment of the circle in which the arrow on the wheel stopped.

The lottery pairs in Appendix 1 contain 28 common consequence or common ratio effects, or combinations of both effects (see bottom of the table in the appendix). A common consequence or common ratio effect involves two lottery pairs such that an EU maximiser either chooses the safe lottery in both pairs or the risky lottery in both pairs. In contrast, it is commonly observed in experiments that many subjects choose the safe lottery in one pair and the risky lottery in the other pair, a choice pattern violating EU.

Since sessions D and E elicited certainty equivalents, our analysis will only rely on the data of sessions A, B, and C. In these sessions the single lottery pairs appeared in randomised order on the screen and subjects had to indicate whether they prefer the left lottery or the right lottery or whether they are indifferent. After pressing the corresponding key the choice had to be confirmed by pressing the return key. If a question of sessions A, B, or C was selected for the reward of a subject, she or he

could simply play out the chosen lottery. In the case of indifference, one lottery of the pair was chosen by the experimenter.

3 The Hypothesis

Our theoretical framework is based on the theory of errors developed by Hey (1995). In this theory individuals are assumed to have deterministic preferences between lotteries which can be represented by a functional V where $V(S, R) > 0$ (< 0) indicates that lottery S is strictly preferred (strictly not preferred) to lottery R . However, individuals sometimes make errors such that the actual choice may not correspond to the given preference relation. Formally, there is a stochastic error term ε such that in practice the value of $V(S, R) + \varepsilon$ determines the choice between S and R . More precisely, the individual will choose S (R) if and only if $V(S, R) + \varepsilon > 0$ (< 0). It is assumed that ε has a median of zero which implies that the actual choice is contrary to the true preferences with a probability of less than 50%. For simplicity one could additionally assume that ε is symmetrically distributed around a mean of zero; such an assumption is, however, not necessary for the present analysis. We do not assume that ε is identical for all lottery pairs, which implies that the constant error model of Harless and Camerer (1994) can be constructed as a special case of our framework. In the following, a lottery S will be represented by a vector $S = (x_1, s_1; x_2, s_2; \dots; x_n, s_n)$ indicating that consequence x_i has probability s_i . Contrary to Hey (1995), we assume that preferences are always in accordance with EU. Hence, there exists a von Neumann-Morgenstern utility function u such that $V(S, R)$ equals the difference between the EU of S and the EU of R , i.e. $V(S, R) = \sum_i u(x_i)(s_i - r_i)$.

Recall from the preceding section that the common consequence effect and the common ratio effect involves two lottery pairs (S, R) and (S^*, R^*) such that each EU maximiser prefers the safe lottery S over the risky lottery R in the first lottery pair if and only if she or he prefers S^* to R^* in the second one. In terms of our model this gives $V(S, R) \geq 0$ if and only if $V(S^*, R^*) \geq 0$ for all functions u . In the experiment we have six observations for each problem: the three choices from the first lottery pair in sessions A, B, and C and the three choices from the second lottery pair in sessions A, B, and C. In the following, these observations will be represented by a vector where the first three entries report the choices from the first lottery pair in sessions A, B, and C respectively, while the last

three entries report the choices from the second lottery pair in sessions A, B, and C respectively. Hence, for instance (S, R, S, R*, R*, S*) indicates that a subject chose S over R in sessions A and C, R over S in session B, R* over S* in sessions A and B, and S* over R* in session C. In the following, we say a given problem satisfies *repetition-consistency* if the first three entries are identical and the last three entries are identical for this problem, i.e. we have (S, S, S, S*, S*, S*), (R, R, R, R*, R*, R*), (S, S, S, R*, R*, R*), or (R, R, R, S*, S*, S*). In other words, repetition-consistency for a given choice problem means that the three choices for the first lottery pair are identical and the three choices for the second lottery pair are identical, but it does not necessarily mean that choices are in accordance with EU. Conversely, repetition-consistency is violated if the individual made contradictory choices in the single sessions for at least one of the two lottery pairs, which we call *repetition-inconsistency*. Besides repetition-consistency, the second important concept is given by *choice-errors*. A choice-error occurs if one given choice in any session and for any lottery pair deviates from true preferences, for example, a subject chose R over S although $V(S, R) > 0$. Note that the probability of a choice-error is always less than 50% since the median of ε has been assumed to be equal to zero. Assuming that EU is the correct model, a violation of EU can obviously not occur in the absence of a choice-error. But it can occur without a violation of repetition-consistency, if the same choice-error is committed for one lottery pair three times in row.

Let us assume that according to true preferences, S is better than R and, consequently, S* is better than R* since true preferences are assumed to be consistent with EU. Moreover, assume that the probability of a choice-error is α when choosing between S and R and β when choosing between S* and R*. This implies that (S, S, S, S*, S*, S*) is observed with a probability of $(1-\alpha)^3(1-\beta)^3$, (R, R, R, R*, R*, R*) with a probability of $\alpha^3\beta^3$, (S, S, S, R*, R*, R*) with a probability of $(1-\alpha)^3\beta^3$ and (R, R, R, S*, S*, S*) with a probability of $\alpha^3(1-\beta)^3$. Since these are the only four choice patterns satisfying repetition-consistency, this easily allows us (see appendix 2) to calculate the probability of a violation of EU in the case of repetition-consistency, which will be referred to as p_{rc} . Analogously, we calculate in the appendix the probability of a violation of EU in the case of repetition-inconsistencies, which is called p_{ri} .

It turns out that according to our model, violations of EU have a higher probability in the cases of repetition-inconsistency than in the cases of repetition-

consistency, i.e. for any given $\alpha, \beta > 0$ we have $p_{ri} - p_{rc} > 0$. Precisely this implication of the model will be tested in the experiment. Table 1 reports this difference for varying values of α and β . Suppose, for instance, that $\alpha = \beta = 0.2$, then the probability of a violation of EU equals 0.03 in case of repetition-consistency (see Table A2 in appendix 2), while it equals 0.43 in cases of repetition-inconsistency (see Table A3 in appendix 2). This yields a difference of 40 percentage points. Table 1 shows that, in accordance with our hypothesis, this difference between p_{ri} and p_{rc} is always positive if, as implied by our model, α and β are less than 0.5.

Table 1: The difference between p_{ri} and p_{rc}

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	-	0.35	0.37	0.38	0.38	0.38	0.36	0.31	0.24	0.13	0
	0.05	0.35	0.36	0.37	0.38	0.39	0.38	0.36	0.32	0.24	0.13	0
	0.10	0.37	0.37	0.38	0.39	0.39	0.39	0.36	0.32	0.24	0.13	0
	0.15	0.38	0.38	0.39	0.39	0.4	0.39	0.37	0.32	0.24	0.13	0
	0.20	0.38	0.39	0.39	0.4	0.4	0.39	0.36	0.32	0.24	0.13	0
	0.25	0.38	0.38	0.39	0.39	0.39	0.38	0.35	0.31	0.23	0.12	0
	0.30	0.36	0.36	0.36	0.37	0.36	0.35	0.33	0.29	0.21	0.12	0
	0.35	0.31	0.32	0.32	0.32	0.32	0.31	0.29	0.25	0.18	0.1	0
	0.40	0.24	0.24	0.24	0.24	0.24	0.23	0.21	0.18	0.14	0.07	0
	0.45	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.1	0.07	0.04	0
	0.50	0	0	0	0	0	0	0	0	0	0	0

So far we have verified that in our model, for S preferred to R, violations of EU have a higher probability in the case of repetition-inconsistency than in the case of repetition-consistency. Let us call this result the hypothesis. Recall that ε and therefore also α and β were not assumed to be identical for all choice problems. Moreover, note that in the analysis so far the values of α and β were completely arbitrary, i.e. the hypothesis is valid for all values of α and β less than 0.5. This implies that the hypothesis is valid within our framework for all choice problems and all subjects as long as S is preferred to R. What happens if R is preferred to S (and therefore also R^* to S^*)? It turns out (see appendix) that in this case the probabilities of violations of EU, p_{rc} and p_{ri} , are completely identical to the case S preferred to R, i.e. the hypothesis is also valid for this case. Overall, we can therefore conclude that, independently of particular preferences, our model of EU plus error term implies that violations of EU have, for all subjects and all choice problems, a higher probability in cases of repetition-inconsistency. This result will be tested by our experiment. However, our design can go one step further since it does not only allow a test of the implications of EU plus error term, but it also allows, in

some sense, a discrimination between EU and non-expected utility (NEU). Consider any arbitrary NEU model which implies, in the absence of choice-errors, a violation of EU for a particular choice problem, i.e. according to true preferences, S is preferred to R and R* is preferred to S* (or alternatively, R is preferred to S and S* is preferred to R*). As we show in the appendix, the error model has the opposite implication than EU for these preferences, i.e. violations of EU have a lower probability in cases of repetition-inconsistency. Thus, our test does not only allow one to analyse the implications of EU but also to compare the implications of EU and NEU, at least for NEU preferences which violate EU for the considered choice problem.

Before presenting the experimental results, let us clarify some points of the data analysis. Note that in the case of repetition-inconsistency we may observe violations of EU in some sessions while in others we do not. Suppose that we have for a given subject and a given choice problem the response pattern (S, S, R, S*, R*, R*). Here, we have a violation of EU in session B while the choices in sessions A and C are consistent with EU. Consequently, this pattern will be regarded as a 33.33% violation of EU in the statistical analysis of the next section. Analogously, (S, S, S, S*, R*, R*) has to be regarded as a 66.67% violation, (S, S, S, R*, R*, R*) as a 100% violation, and (S, S, S, S*, S*, S*) as a 0% violation.

Finally, it should be explained how indifference has been treated in the analysis. If for a given lottery pair and a given subject there was one indifference in the three sessions this indifference has been treated as a missing observation. If there were two or three cases of indifference, the complete choice problem for this subject has been removed from the analysis. This procedure allowed us to exclude violations of EU which simply result from imprecise preference.

4 Results

(i) Overall Data

Let us first give an overview of our results and consider the complete sample of all subjects and all lottery pairs (observed choice behaviour for every single lottery pair is reported in appendix 1). In this sample the overall violation rate of EU is 23.44%. Restricting attention to cases of repetition-consistency, this violation rate decreases to

12.45%; whereas, it increases to 45.03% in the cases of repetition-inconsistency. This is consistent with the main hypothesis of this paper and provides support for EU, at least within a framework which takes choice-errors into account.

(ii) Between-subject Analysis

Let us now turn to the between-subject analysis. Table 2 records for each choice problem the average violation rate of EU for repetition-inconsistent choice (second column) and for repetition-consistent choice (third column). Additionally, the second and third columns report in brackets the number of subjects in the respective category, i.e. the number of independent observations. Where the number of subjects for a given choice problem does not add up to 24, subjects were excluded because they stated indifference at least twice in one involved lottery pair (see end of the preceding section). The last column of Table 2 reports the difference between these violation rates. A positive difference indicates a higher violation rate for repetition-inconsistent choice than for repetition-consistent choice, while a negative difference indicates the opposite. The size of the difference is tested by a two-tailed Wilcoxon rank sum test. Significant test results are also reported in the last column of Table 2 with asterisks (right-hand side); one character indicates a significance level of 10%, two characters of 5%, three characters of 1%, and no character indicates insignificance at 10%. At a significance level of 5%, the test results suggest that the exclusion of repetition-inconsistent choice would lead to a significant reduction of the violation rate for 20 of the 28 choice problems and in no choice problem to a significant increase. In other words, our hypothesis is supported for 20 choice problems while the contrary is not supported for any choice problem. In sum, the between-subject analysis confirms the inferences from the overall data for both the common consequence and the common ratio effect.

(iii) Within-subject Analysis

Let us now turn to the within-subject analysis. Table 3 records the average violation rate of EU for repetition-consistent choice problems (left-hand side of second column) and for the repetition-inconsistent choice problems (left-hand side of third column) for each subject; the number of observations are reported in brackets. Where these observations do not add up to 28, some choice problems had to be excluded since the subject indicated indifference in more than one of the involved lottery pairs. The

Table 2: Violation rates for each choice problem across all subjects

Choice problem	Repetition-Inconsistency [#]	Repetition-Consistency [#]	Difference	Significance level ^{a)}
1, 2	69.05% [7]	18.18% [11]	51%	**
1, 3	53.33% [5]	56.25% [16]	-3%	
1, 4	47.92% [8]	26.67% [15]	21%	*
2, 3	26.67% [5]	25.00% [12]	2%	
2, 4	43.75% [8]	33.33% [9]	10%	
3, 4	63.89% [6]	26.67% [15]	37%	*
5, 6	33.33% [6]	5.56% [18]	28%	***
5, 7	30.56% [6]	0.00% [16]	31%	***
6, 7	39.58% [8]	7.14% [14]	32%	***
8, 9	50.00% [10]	55.56% [9]	-6%	
8,10	48.61% [12]	60.00% [10]	-11%	
9,10	50.00% [10]	11.11% [9]	39%	***
11,12	53.33% [10]	0.00% [11]	53%	***
13,14	47.22% [6]	0.00% [14]	47%	***
15,16	45.83% [4]	0.00% [15]	46%	***
17,18	22.22% [6]	5.56% [18]	17%	**
17,19	27.78% [6]	0.00% [18]	28%	***
17,20	29.17% [8]	0.00% [15]	29%	***
18,19	46.67% [5]	5.26% [19]	41%	**
18,20	47.62% [7]	6.25% [16]	41%	***
19,20	33.33% [4]	0.00% [19]	33%	**
21,22	37.50% [8]	7.69% [13]	30%	***
21,23	41.67% [8]	15.38% [13]	26%	**
22,23	28.57% [7]	15.38% [13]	13%	
24,25	38.89% [6]	6.25% [16]	33%	***
26,27	48.81% [14]	0.00% [8]	49%	***
26,28	54.17% [8]	14.29% [14]	40%	***
27,28	58.97% [13]	0.00% [9]	59%	***

a) *** 1%, ** 5%, * 10 % (significance level, two-tailed Wilcoxon rank sum test).

difference in the violation rates between time-inconsistent and time-consistent choice is recorded in the last column of Table 3. A positive difference indicates a higher violation rate for repetition-inconsistent choice than for repetition-consistent choice, a negative difference indicates the opposite, and a missing observation indicates subjects whose choices were always repetition-consistent. Table 3 shows that the exclusion of repetition-inconsistent choice leads to a reduction of the violation rate for 20 out of 21 subjects without missing observations. The reduction of violations is significant at 1% when one moves from repetition-inconsistent to repetition-consistent choice as a Wilcoxon signed ranks test shows. Therefore, also the within-subject analysis supports the research hypothesis of the present paper.

(iv) Analysis of Choice-Errors

We can also use our data in order to get some insights into the occurrence of choice-errors. More precisely, we will set the observed violation rates in the case of repetition-consistency equal to p_{rc} and the observed violation rate in the case of repetition-inconsistency equal to p_{ri} . Doing this enables us to calculate α and β in both cases with the help of equations (1) and (2) in the appendix. In other words, we can derive the implication of our data for the frequency of choice-errors. Additionally, we can check whether the derived values of α and β are similar for the cases of repetition-inconsistency and those of repetition-consistency or whether there are systematic differences which could be regarded as evidence against our model.

For simplicity we will rely on a constant error model in this analysis, i.e. we assume that the probability of a choice-error is identical for a given subject in each lottery pair, which implies $\alpha = \beta$. Without this assumption one could only derive a continuum of admissible α and β combinations for each observation. However, we do not assume that the probability of errors is identical for different subjects.

Let us first mention that our model implies some restrictions on the values of p_{rc} and p_{ri} if α and β are assumed to be greater than zero and less than 0.5. Tables A2-A5 in the appendix show that we must have $0 \leq p_{rc} \leq 0.5$ and $0.33 \leq p_{ri} \leq 0.5$ in the case of EU as well as $0.5 \leq p_{rc} \leq 1$ and $0.5 \leq p_{ri} \leq 0.67$ in the case of a NEU model which

Table 3: Violation rates and choice-error approximations for each subject

Subject ID	Repetition-Inconsistency		Repetition-Consistency		Difference
	Violation rate [#]	Approximation α, β	Violation rate [#]	Approximation α, β	
#1	66.67% [4]	-	22.22% [18]	34.49%	44%
#2	- [0]	-	0.00% [8]	0.00%	-
#3	53.13% [16]	-	0.00% [11]	0.00%	53%
#4	42.59% [9]	19.65%	26.32% [19]	36.29%	16%
#5	44.05% [14]	23.21%	21.43% [14]	34.12%	23%
#6	50.00% [6]	50.00%	18.18% [22]	32.56%	32%
#7	26.67% [15]	-	33.33% [12]	39.20%	-7%
#8	44.44% [21]	24.20%	0.00% [7]	0.00%	44%
#9	38.19% [24]	9.93%	0.00% [2]	0.00%	38%
#10	38.10% [7]	9.74%	28.57% [21]	37.23%	10%
#11	- [0]	-	7.14% [14]	25.24%	-
#12	60.00% [5]	-	29.41% [17]	37.58%	31%
#13	44.44% [3]	24.20%	0.00% [25]	0.00%	44%
#14	54.17% [8]	-	0.00% [20]	0.00%	54%
#15	50.00% [6]	50.00%	0.00% [20]	0.00%	50%
#16	41.67% [2]	17.52%	9.09% [11]	26.94%	33%
#17	57.14% [21]	-	0.00% [7]	0.00%	57%
#18	45.83% [4]	27.97%	29.17% [24]	37.48%	17%
#19	33.33% [8]	0.00%	29.41% [17]	37.58%	4%
#20	- [0]	-	0.00% [26]	0.00%	-
#21	33.33% [2]	0.00%	0.00% [26]	0.00%	33%
#22	33.33% [3]	0.00%	0.00% [23]	0.00%	33%
#23	42.86% [14]	20.29%	33.33% [12]	39.20%	10%
#24	45.61% [19]	27.35%	11.11% [9]	28.44%	35%
Average	45.03% [8.79]	20.27%	12.45% [16.04]	18.60%	33%***

*** 1% significant (two-tailed Wilcoxon signed ranks test); - missing observation

implies a violation of EU for the considered choice problem. (The numbers 0.33 in the case of EU and 0.67 in the case of NEU can not be taken directly from the tables since they result from strictly positive values of α and β less than 0.01). Comparing these numbers with the observed violation rates for the single subjects in Table 3 yields the

following results: (i) in the case of repetition-consistency all violation rates are consistent with the predictions of EU; whereas, no violation rate is consistent with NEU; (ii) in the case of repetition-inconsistency 15 out of 21 violation rates without missing observations are consistent with the predictions of EU; whereas, only 7 are consistent with NEU.

Assuming EU, we have now calculated the implied relative frequencies of choice-errors (i.e. $\alpha = \beta$) for the subjects with consistent violation rates. The results are stated on the right-hand sides of the second and third columns of Table 3. Although there exist substantial differences for these values in the cases of repetition-inconsistency and those of repetition-consistency for a number of subjects, there does not exist a systematic pattern and the average values (see last row of Table 3) are quite close (0.1860 and 0.2027). Moreover, the difference between these average values is insignificant at any reasonable significance level as the Wilcoxon signed ranks test indicates ($p = .861$).

Overall, an error rate of ca. 20% seems to be reasonable (comparable numbers have been reported by Harless and Camerer (1994) and Birnbaum and Bahra (2005)) and the results of the present subsection do not reveal systematic deviations from the predictions of our model.

5 Conclusions

This paper presented an experimental analysis of individual choice under risk in the presence of errors. Overall, our results show that the main implication of representing preferences by EU plus error term is supported by our data. Before closing, we should comment on one issue which has often been put forward as an argument against representing preferences by EU plus error term. Recall that violations of EU in common consequence and common ratio effects are not random deviations but appear to be highly systematic. For instance, in the classical common ratio problem formed by lottery pairs one and two (see Table 1, i.e. S offers a 100% chance of £30; R offers a 80% chance of £40 and a 20% chance of £0; S* offers a 25% chance of £30 and a 75% chance of £0; R* offers a 20% chance of £40 and a 80% chance of £0) the usual

pattern of violations is the choice of S and R*; whereas, the choice of R and S* is very rarely observed (this is also true for our data). Such highly systematic violations seem, at first glance, to contradict a model such as ours that attributes violations of EU to errors. However, there is no contradiction. Instead, an obvious reason for systematic violations may be an asymmetric error term. But also with symmetric error terms, systematic violations of EU can be explained in the present model. Suppose we have $u(\pounds 0) = 0$, $u(\pounds 30) = 30$, and $u(\pounds 40) = 35$. This yields for the common ratio problem $V(S, R) = 30 - 0.8 \cdot 35 = 2$ and $V(S^*, R^*) = 0.5$. If the error term is uniformly distributed between -1 and 1, we can never observe the choice of R and S*; whereas, the choice of S and R* is observed in 25% of the cases. Consequently, our model is consistent with systematic deviations of EU even in the presence of a symmetric error term. We should, however, add that the main goal of the present paper was not to analyse specific hypotheses concerning the error term but to investigate whether the fundamental implications of EU plus error term are broadly confirmed or not.

Appendix 1: The Lottery Pairs

Table A1: The Lottery Pairs

No.	Safe Lottery				Risky Lottery				Overall Choice (in %)		
	£0	£10	£30	£40	£0	£10	£30	£40	Safe	Indifferent	Risky
1	0.00	0.00	1.00	0.00	0.20	0.00	0.00	0.80	69.4	0.0	30.6
2	0.75	0.00	0.25	0.00	0.80	0.00	0.00	0.20	25.0	20.8	54.2
3	0.30	0.60	0.10	0.00	0.32	0.60	0.00	0.08	8.3	16.7	75.0
4	0.00	0.60	0.10	0.30	0.02	0.60	0.00	0.38	40.3	8.3	51.4
5	0.00	1.00	0.00	0.00	0.70	0.00	0.00	0.30	65.3	0.0	34.7
6	0.00	0.50	0.50	0.00	0.35	0.00	0.50	0.15	66.7	0.0	33.3
7	0.50	0.50	0.00	0.00	0.85	0.00	0.00	0.15	52.8	8.3	38.9
8	0.00	0.00	0.70	0.30	0.15	0.00	0.00	0.85	65.3	1.4	33.3
9	0.80	0.00	0.14	0.06	0.83	0.00	0.00	0.17	27.8	19.4	52.8
10	0.20	0.00	0.74	0.06	0.23	0.00	0.60	0.17	18.1	5.6	76.4
11	0.00	0.20	0.80	0.00	0.00	0.50	0.00	0.50	68.1	4.2	27.8
12	0.50	0.10	0.40	0.00	0.50	0.25	0.00	0.25	66.7	13.9	19.4
13	0.00	0.20	0.60	0.20	0.20	0.00	0.40	0.40	65.3	16.7	18.1
14	0.00	0.10	0.30	0.60	0.10	0.00	0.20	0.70	69.4	13.9	16.7
15	0.20	0.80	0.00	0.00	0.80	0.00	0.00	0.20	68.1	19.4	12.5
16	0.10	0.40	0.50	0.00	0.40	0.00	0.50	0.10	72.2	18.1	9.7
17	0.00	0.40	0.60	0.00	0.40	0.00	0.00	0.60	59.7	1.4	38.9
18	0.50	0.20	0.30	0.00	0.70	0.00	0.00	0.30	70.8	1.4	27.8
19	0.00	0.20	0.30	0.50	0.20	0.00	0.00	0.80	56.9	2.8	40.3
20	0.00	0.20	0.70	0.10	0.20	0.00	0.40	0.40	62.5	2.8	34.7
21	0.00	0.00	0.50	0.50	0.10	0.00	0.00	0.90	56.9	1.4	41.7
22	0.50	0.00	0.50	0.00	0.60	0.00	0.00	0.40	43.1	12.5	44.4
23	0.25	0.50	0.25	0.00	0.30	0.50	0.00	0.20	26.4	16.7	56.9
24	0.00	0.50	0.00	0.50	0.20	0.20	0.00	0.60	63.9	0.0	36.1
25	0.50	0.25	0.00	0.25	0.60	0.10	0.00	0.30	56.9	8.3	34.7
26	0.00	0.25	0.50	0.25	0.00	0.35	0.00	0.65	11.1	9.7	79.2
27	0.00	0.00	0.75	0.25	0.00	0.10	0.25	0.65	29.2	5.6	65.3
28	0.25	0.25	0.50	0.00	0.25	0.35	0.00	0.40	20.8	9.7	69.4

Note: common consequence effects: (3,4), (6,7), (9,10), (18,19), (18,20), (19,20), (21,22), (26,27), (26,28), (27,28); common ratio effects: (1,2), (5,7), (8,9), (11,12), (17,18), (24,25); combinations of common ratio and common consequence effect: (1,3), (1,4), (2,3), (2,4), (5,6), (8,10), (13,14), (15,16), (17,19), (17,20), (21,23), (22,23).

Appendix 2: The Main Hypothesis

In the following, we derive the main hypothesis (what is the main or research hypothesis of the paper? – Please state here) of the paper assuming that true

preferences satisfy EU. Therefore, we first calculate for one given choice problem the probabilities of a violation of EU (i) for cases of repetition-inconsistency and (ii) for cases of repetition-consistency. Initially, we assume that subjects prefer S over R according to true preferences and consequently S* over R*. (Later we will consider the opposite preferences). Table A2 records all possible response patterns of subjects and the resulting relative frequency of a violation of EU. In the fifth row, for instance, the subject chooses S and S* in session A (consistent with EU) but violates EU in sessions B and C by choosing S and R*. Consequently, we observe a violation of EU in two out of three cases. Recalling that S is preferred to R according to true preferences (and therefore also S* to R*) we obtain the probability of each choice pattern stated in the third column (recall that the subject chooses R by mistake with a probability of α and R* with a probability of β). Finally, the fourth column indicates whether the response pattern contains a repetition-inconsistency (i.e. choice between S and R or S* and R* is not identical in all three sessions) or not.

Table A2: Possible Response Patterns

Observation	Relative frequency of a violation of EU	Probability of observation	Repetition-Inconsistency
S, S, S, S*, S*, S*	0	$(1-\alpha)^3(1-\beta)^3$	No
S, S, S, S*, S*, R*	1/3	$(1-\alpha)^3(1-\beta)^2\beta$	Yes
S, S, S, S*, R*, S*			
S, S, S, R*, S*, S*			
S, S, S, S*, R*, R*	2/3	$(1-\alpha)^3(1-\beta)\beta^2$	Yes
S, S, S, R*, S*, R*			
S, S, S, R*, R*, S*			
S, S, S, R*, R*, R*	1	$(1-\alpha)^3\beta^3$	No
S, S, R, S*, S*, S*	1/3	$(1-\alpha)^2\alpha(1-\beta)^3$	Yes
S, S, R, S*, S*, R*	0	$(1-\alpha)^2\alpha(1-\beta)^2\beta$	Yes
S, S, R, S*, R*, S*			
S, S, R, R*, S*, S*			
S, S, R, S*, R*, R*	1/3	$(1-\alpha)^2\alpha(1-\beta)\beta^2$	Yes
S, S, R, R*, S*, R*	1/3		
S, S, R, R*, R*, S*	1		
S, S, R, R*, R*, R*	2/3	$(1-\alpha)^2\alpha\beta^3$	Yes
S, R, S, S*, S*, S*	1/3	$(1-\alpha)^2\alpha(1-\beta)^3$	Yes
S, R, S, S*, S*, R*	2/3	$(1-\alpha)^2\alpha(1-\beta)^2\beta$	Yes
S, R, S, S*, R*, S*			
S, R, S, R*, S*, S*			0
S, R, S, S*, R*, R*	1/3	$(1-\alpha)^2\alpha(1-\beta)\beta^2$	Yes
S, R, S, R*, S*, R*	1		
S, R, S, R*, R*, S*	1/3		
S, R, S, R*, R*, R*	2/3	$(1-\alpha)^2\alpha\beta^3$	Yes
R, S, S, S*, S*, S*	1/3	$(1-\alpha)^2\alpha(1-\beta)^3$	Yes
R, S, S, S*, S*, R*	2/3	$(1-\alpha)^2\alpha(1-\beta)^2\beta$	Yes
R, S, S, S*, R*, S*			
R, S, S, R*, S*, S*			0
R, S, S, S*, R*, R*	1	$(1-\alpha)^2\alpha(1-\beta)\beta^2$	Yes
R, S, S, R*, S*, R*	1/3		
R, S, S, R*, R*, S*	1/3		

R, S, S, R*, R*, R*	2/3	$(1-\alpha)^2\alpha\beta^3$	Yes
S, R, R, S*, S*, S*	2/3	$(1-\alpha)\alpha^2(1-\beta)^3$	Yes
S, R, R, S*, S*, R*	1/3	$(1-\alpha)\alpha^2(1-\beta)^2\beta$	Yes
S, R, R, S*, R*, S*	1/3		
S, R, R, R*, S*, S*	1		
S, R, R, S*, R*, R*	0	$(1-\alpha)\alpha^2(1-\beta)\beta^2$	Yes
S, R, R, R*, S*, R*	2/3		
S, R, R, R*, R*, S*	2/3		
S, R, R, R*, R*, R*	1/3	$(1-\alpha)\alpha^2\beta^3$	Yes
R, S, R, S*, S*, S*	2/3	$(1-\alpha)\alpha^2(1-\beta)^3$	Yes
R, S, R, S*, S*, R*	1/3	$(1-\alpha)\alpha^2(1-\beta)^2\beta$	Yes
R, S, R, S*, R*, S*	1		
R, S, R, R*, S*, S*	1/3		
R, S, R, S*, R*, R*	2/3	$(1-\alpha)\alpha^2(1-\beta)\beta^2$	Yes
R, S, R, R*, S*, R*	0		
R, S, R, R*, R*, S*	2/3		
R, S, R, R*, R*, R*	1/3	$(1-\alpha)\alpha^2\beta^3$	Yes
R, R, S, S*, S*, S*	2/3	$(1-\alpha)\alpha^2(1-\beta)^3$	Yes
R, R, S, S*, S*, R*	1	$(1-\alpha)\alpha^2(1-\beta)^2\beta$	Yes
R, R, S, S*, R*, S*	1/3		
R, R, S, R*, S*, S*	1/3		
R, R, S, S*, R*, R*	2/3	$(1-\alpha)\alpha^2(1-\beta)\beta^2$	Yes
R, R, S, R*, S*, R*	2/3		
R, R, S, R*, R*, S*	0		
R, R, S, R*, R*, R*	1/3	$(1-\alpha)\alpha^2\beta^3$	Yes
R, R, R, S*, S*, S*	1	$\alpha^3(1-\beta)^3$	No
R, R, R, S*, S*, R*	2/3	$\alpha^3(1-\beta)^2\beta$	Yes
R, R, R, S*, R*, S*	2/3		
R, R, R, R*, S*, S*	2/3		
R, R, R, S*, R*, R*	1/3	$\alpha^3(1-\beta)\beta^2$	Yes
R, R, R, R*, S*, R*	1/3		
R, R, R, R*, R*, S*	1/3		
R, R, R, R*, R*, R*	0	$\alpha^3\beta^3$	No

From Table A2 we can now calculate the probabilities. The probability of a violation of EU in cases of repetition-consistency is determined by

$$(1) \quad p_{rc} = \frac{(1-\alpha)^3\beta^3 + \alpha^3(1-\beta)^3}{(1-\alpha)^3(1-\beta)^3 + \alpha^3\beta^3 + (1-\alpha)^3\beta^3 + \alpha^3(1-\beta)^3}.$$

The resulting values of this probability for varying values of α and β are reported in Table A3.

Table A3: Probability of violations of EU in the case of repetition-consistency (p_{rc})

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.05	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.10	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.15	0.01	0.01	0.01	0.01	0.02	0.04	0.08	0.14	0.23	0.36	0.5
	0.20	0.02	0.02	0.02	0.02	0.03	0.05	0.09	0.15	0.24	0.36	0.5
	0.25	0.04	0.04	0.04	0.04	0.05	0.07	0.1	0.16	0.25	0.36	0.5
	0.30	0.07	0.07	0.07	0.08	0.09	0.1	0.14	0.19	0.27	0.38	0.5
	0.35	0.14	0.14	0.14	0.14	0.15	0.16	0.19	0.23	0.3	0.39	0.5
	0.40	0.23	0.23	0.23	0.23	0.24	0.25	0.27	0.3	0.35	0.42	0.5
	0.45	0.35	0.35	0.35	0.36	0.36	0.36	0.38	0.39	0.42	0.46	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

From Table A3 we can also calculate the probability of violations of EU in cases of repetition-inconsistency. This probability is determined by

$$p_{ri} = \frac{[(1-\alpha)^3(1-\beta)^3\beta + 2(1-\alpha)^3(1-\beta)\beta^2 + (1-\alpha)^2\alpha(1-\beta)^3 + 4(1-\alpha)^2\alpha(1-\beta)^2\beta + 5(1-\alpha)^2\alpha(1-\beta)\beta^2 + 2(1-\alpha)^2\alpha\beta^3 + 2(1-\alpha)\alpha^2(1-\beta)^3 + 5(1-\alpha)\alpha^2(1-\beta)^2\beta + 4(1-\alpha)\alpha^2(1-\beta)\beta^2 + (1-\alpha)\alpha^2\beta^3 + 2\alpha^3(1-\beta)^2\beta + \alpha^3(1-\beta)\beta^2]}{[1 - [(1-\alpha)^3(1-\beta)^3 + \alpha^3\beta^3 + (1-\alpha)^3\beta^3 + \alpha^3(1-\beta)^3]]}$$

and the resulting values of this probability for varying values of α and β are reported in Table A4.

Table A4: Probability of violations of EU in the case of repetition-inconsistency (p_{ri})

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	-	0.35	0.37	0.38	0.4	0.42	0.43	0.45	0.47	0.48	0.5
	0.05	0.35	0.36	0.37	0.39	0.4	0.42	0.43	0.45	0.47	0.48	0.5
	0.10	0.37	0.37	0.38	0.39	0.41	0.42	0.44	0.45	0.47	0.48	0.5
	0.15	0.38	0.39	0.39	0.41	0.42	0.43	0.44	0.46	0.47	0.49	0.5
	0.20	0.4	0.4	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.49	0.5
	0.25	0.42	0.42	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
	0.30	0.43	0.43	0.44	0.44	0.45	0.46	0.47	0.47	0.48	0.49	0.5
	0.35	0.45	0.45	0.45	0.46	0.46	0.47	0.47	0.48	0.49	0.49	0.5
	0.40	0.47	0.47	0.47	0.47	0.47	0.48	0.48	0.49	0.49	0.5	0.5
	0.45	0.48	0.48	0.48	0.49	0.49	0.49	0.49	0.49	0.5	0.5	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

We took the inequality $p_{rc} \geq p_{ri}$ and verified with the “Maple” software that, under the restriction $0 < \alpha, \beta < 0.5$, values of α and β do not exist such that this inequality is satisfied. Additionally, we calculated the difference between p_{ri} and p_{rc} for varying values of α and β . We considered a rather high number of values of α and β between zero and 0.5 by starting from $\alpha = \beta = 0$ and increasing α and β in 0.0001 steps. A shortened overview of our calculations is given in Table 1 in section 3 which shows that the difference is always positive in accordance with our hypothesis.

So far we have shown that our hypothesis is true for one subject preferring S to R and S* to R* in a given choice problem. Since the difference between p_{ri} and p_{rc} is, however, positive for all admissible values of α and β we can conclude that the hypothesis is true for all subjects and all choice problems as long as S and S* are preferred.

Recall that the probabilities of observing a choice of S, R, S*, or R* are $1 - \alpha$, α , $1 - \beta$, and β , respectively, given our initial assumption that S and S* are preferred choices. Now suppose that, according to true preferences, R is preferred to S and R* to S*. In this case, the probabilities of observing a choice of S, R, S*, or R* are α , $1 - \alpha$, β , and $1 - \beta$, respectively. Consequently, all values of α in Table A1 and also in the equations of p_{rc} and p_{ri} have to be replaced by $1 - \alpha$, while all values of β have to be replaced by $1 - \beta$ if R and R* are the preferred choices. However, it turns out that p_{rc} and p_{ri} remain unchanged if all values of α are replaced by $1 - \alpha$ and all values of β are replaced by $1 - \beta$. Since p_{rc} and p_{ri} remain unchanged, the difference between p_{ri} and p_{rc} will again be positive for all admissible values of α and β , which means that the hypothesis is also valid if R and R* are preferred.

Let us now consider NEU preferences which violate EU for the considered choice problem. Recall again that the probabilities of observing a choice of S, R, S*, or R* are $1 - \alpha$, α , $1 - \beta$, and β , respectively, if S and S* are preferred. Now suppose that a subject violates EU by preferring S and R* (or R and S*). In this case, the probabilities of observing a choice of S, R, S*, or R* are $1 - \alpha$, α , β , and $1 - \beta$, respectively (or α , $1 - \alpha$, $1 - \beta$, and β , respectively, if R and S* are preferred). This means that in Table A1 and in the equations of p_{rc} and p_{ri} either β has to be replaced by $1 - \beta$ or α has to be replaced by $1 - \alpha$. For both cases we calculated p_{rc} (see Table A4), and p_{ri} (see Table A5) and it turned out that both cases yield identical values of p_{ri} and identical values of p_{rc} . We also calculated the difference between both probabilities for a high number of values of α and β by varying α and β in 0.0001 steps. A shortened overview of the calculations is presented in Table A6. It turns out that the difference is always negative, i.e. we have the opposite implication than in the case of EU.

Table A4: p_{rc} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.05	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.10	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.15	0.99	0.99	0.99	0.99	0.98	0.96	0.92	0.86	0.77	0.64	0.5
	0.20	0.98	0.98	0.98	0.98	0.97	0.95	0.91	0.85	0.76	0.64	0.5
	0.25	0.96	0.96	0.96	0.96	0.95	0.93	0.9	0.84	0.75	0.64	0.5
	0.30	0.93	0.93	0.93	0.92	0.91	0.9	0.86	0.81	0.73	0.62	0.5
	0.35	0.86	0.86	0.86	0.86	0.85	0.84	0.81	0.77	0.7	0.61	0.5
	0.40	0.77	0.77	0.77	0.77	0.76	0.75	0.73	0.7	0.65	0.58	0.5
	0.45	0.65	0.65	0.65	0.64	0.64	0.64	0.62	0.61	0.58	0.54	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table A5: p_{ri} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	-	0.65	0.63	0.62	0.6	0.58	0.57	0.55	0.53	0.52	0.5
	0.05	0.65	0.64	0.63	0.61	0.6	0.58	0.57	0.55	0.53	0.52	0.5
	0.10	0.63	0.63	0.62	0.61	0.59	0.58	0.56	0.55	0.53	0.52	0.5
	0.15	0.62	0.61	0.61	0.59	0.58	0.57	0.56	0.54	0.53	0.51	0.5
	0.20	0.6	0.6	0.59	0.58	0.57	0.56	0.55	0.54	0.53	0.51	0.5
	0.25	0.58	0.58	0.58	0.57	0.56	0.55	0.54	0.53	0.52	0.51	0.5
	0.30	0.57	0.57	0.56	0.56	0.55	0.54	0.53	0.53	0.52	0.51	0.5
	0.35	0.55	0.55	0.55	0.54	0.54	0.53	0.53	0.52	0.51	0.51	0.5
	0.40	0.53	0.53	0.53	0.53	0.53	0.52	0.52	0.51	0.51	0.5	0.5
	0.45	0.52	0.52	0.52	0.51	0.51	0.51	0.51	0.51	0.5	0.5	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table A6: The difference between p_{ri} and p_{rc} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
β	0.00	-	-0.35	-0.37	-0.38	-0.38	-0.38	-0.36	-0.31	-0.24	-0.13	0
	0.05	-0.35	-0.36	-0.37	-0.38	-0.39	-0.38	-0.36	-0.32	-0.24	-0.13	0
	0.10	-0.37	-0.37	-0.38	-0.39	-0.39	-0.39	-0.36	-0.32	-0.24	-0.13	0
	0.15	-0.38	-0.38	-0.39	-0.39	-0.4	-0.39	-0.37	-0.32	-0.24	-0.13	0
	0.20	-0.38	-0.39	-0.39	-0.4	-0.4	-0.39	-0.36	-0.32	-0.24	-0.13	0
	0.25	-0.38	-0.38	-0.39	-0.39	-0.39	-0.38	-0.35	-0.31	-0.23	-0.12	0
	0.30	-0.36	-0.36	-0.36	-0.37	-0.36	-0.35	-0.33	-0.29	-0.21	-0.12	0
	0.35	-0.31	-0.32	-0.32	-0.32	-0.32	-0.31	-0.29	-0.25	-0.18	-0.1	0
	0.40	-0.24	-0.24	-0.24	-0.24	-0.24	-0.23	-0.21	-0.18	-0.14	-0.07	0
	0.45	-0.13	-0.13	-0.13	-0.13	-0.13	-0.12	-0.12	-0.1	-0.07	-0.04	0
	0.50	0	0	0	0	0	0	0	0	0	0	0

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